Decomposition Versus Decision-Evaluation

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Decomposition

- Standard approach
- Leads to problems
- Examples:
  – Returns of traded components (Dietz)
  – Performance Attribution (Brinson)
  – Contribution (Everyone)
  – <u>Risk Attribution</u> (Menchero, Hu & Bertrand)

Decomposition
= Top-Down modeling

1. Portfolio level

   Impose economic requirements

   that completely specify.

2. Issue level

   Partially restrict & then

   Rely upon arbitrary assumptions.

Decomposing Returns (Single Day, with Trades)

$$R \equiv \frac{G}{Vo} = \Sigma_j \frac{G_j}{Vo} = \Sigma_j \frac{\mathbf{B_j}}{Vo} \frac{G_j}{\mathbf{B_j}} \equiv \Sigma_j W_j \; R_j$$

1. Meaningful definition of R at the portfolio level.
2. Issue level basis, $\mathbf{B_j}$, can be anything (continuous).
3. Formally Define $R_j \equiv G_j/B_j$ and Require $\Sigma_j W_j = 1$.
4. Link R_j over time → component-level, period returns:

$$R_{j,T} = -1 + \Pi_{t \in T} (1 + R_{j,t})$$

Guesses for Arbitrary Issue Basis

A: Buy & Hold

B: Modified Dietz

C: Purchases in the basis

(Purchases at open & Sales at close)

D: Etc.

A: **Buy & Hold** Basis

Guess: $\mathbf{B_j}$ = Vo_j

Problem: New holdings → $\mathbf{B_j}$ = 0

$W_j = \mathbf{B_j}/Vo = 0$

$R_j = G_j/\mathbf{B_j} = +/- \infty$ (Unbeatable)

B: **Modified Dietz** Basis

Guess: $\mathbf{B_j} = Vo_j + W_T*(Pur_j - Sale_j)$

Problem for long-only portfolio:

$Sale_j > Pur_j + Vo_j/W_T \rightarrow \mathbf{B_j} < 0$

$W_j = \mathbf{B_j}/Vo < 0$

$R_j = G_j/\mathbf{B_j} \rightarrow G_j/R_j < 0$

C: **Purchases in** Basis

Guess: $\mathbf{B_j} = Vo_j + Pur_j$

$\Sigma_j W_j = 1 \rightarrow \mathbf{B_c} = Vo_c - \Sigma_{j \neq c} Pur_j$

Problem for long cash w/ sales fund Pur:

$\Sigma_{j \neq c} Pur_j > Vo_c \rightarrow \mathbf{B_c} < 0$

$W_c = \mathbf{B_c}/Vo < 0$

$R_c = G_c/\mathbf{B_c} \rightarrow G_c/R_c < 0$

ETC.

- Other attempts at guessing the basis in the presence of trades all fail also.
- Standard calculations of

  the return of a traded component

  of a portfolio,

  are incorrect and misleading!

  And to an unknown degree.

FTM

A correct approach to the calculation of the returns of traded components must:

- Avoid Arbitrariness = Ans a Question
- Roll-up unproblematically
- At least for long-only portfolios:

  Disallow

  - W<0
  - R<-100%
- Not the focus of this talk.

Decomposing Active Return Brinson type Attribution

$$\Delta R \equiv R^P - R^B = \text{All} + \text{Sel} + \text{Int} \equiv \Sigma_k X_k$$

Impose Portfolio-level Definition of ΔR

And formal requirement on sum of attributes

And, perhaps, intuitive monotonicity of

- All w/over-weighting good sectors and
- Sel w/over-weighting good issues in sectors

Decomposing Active Return Brinson type Attribution

$\Delta R \equiv R^P - R^B = All' + Sel' + Int' \equiv \Sigma_k X'_k$

$= (All + \mathbf{Ae}) + (Sel + \mathbf{Se}) + (Int + \mathbf{Ie})$

where Ae + Se + Ie = 0. Thus, $\mathbf{X_k}$ **arbitrary!**

Can even still save intuitive monotonicity of

- All' w/over-weighting good sectors and
- Sel' w/over-weighting good issues in sectors

Again,
By fully imposing
Restrictions and Meaning
Only at the Portfolio level,
We do <u>not</u> get valid meaning
at the component (attribute) level.

Contribution

- $R \equiv \Sigma_j W_j \; R_j = \Sigma_j [W_j \; R_j + a_j]$ where $\Sigma_j a_j = 0$.
- $Cont_j \equiv W_j \; R_j$ itself
  Answers NO economic question
  &
  Is arbitrary.

Decomposing Risk-Adjusted Returns

$$IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\Sigma_k \langle X_k \rangle}{\sigma(\Delta R)}$$

$$= \Sigma_k \frac{[\rho(X_k, \Delta R) * \sigma(X_k)]}{\sigma(\Delta R)} * \frac{\langle X_k \rangle}{[\rho(X_k, \Delta R) * \sigma(X_k)]}.$$

- $\equiv \Sigma_k WR_k * IR_K$ where $\rho \equiv$ Correlation $\rightarrow \Sigma_k WR_k = 1$.
- Contribution of Att k to IR = (Risk W of Att)*(IR of Att)

Decomposing Risk-Adjusted Returns

- $IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\Sigma_k \langle X_k \rangle}{\sigma(\Delta R)}$

$$= \Sigma_k \frac{[\rho(X_k, \Delta R) * \sigma(X_k) + E_k]}{\sigma(\Delta R)} * \frac{\langle X_k \rangle}{[\rho(X_k, \Delta R) * \sigma(X_k) + E_k]}.$$

$$\equiv \Sigma_k WR'_k * IR'_K.$$

$\Sigma_k \; E_k = 0 \rightarrow \Sigma_k WR'_k = 1.$

But **IR'$_K$ is Arbitrary!**

Decomposing Risk-Adjusted Returns

$$IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\Sigma_k \langle X_k \rangle}{\sigma(\Delta R)} = \Sigma_k \frac{[\langle X_k \rangle + aa_k]}{\sigma(\Delta R)}$$

□ $\equiv \Sigma_k$ IRCont$_k$, where $\Sigma_k\, aa_k = 0$.

IRCont$_k$ $\equiv$ “The portion of IR attributed to decision k.”

But **IRCont$_K$ is Arbitrary!**

Again,
By fully imposing
Restrictions and Meaning
Only at the Portfolio level,
We do <u>not</u> get valid meaning
at the component (attribute) level.

Properties
(like returns and attributes)
need to be
economically defined
at the level they are employed,
else the arbitrariness breeds
nonsense.

HATS

Top-Down Modeling

The Decomposition of 'Hat Quantity'

HATNESS = #H

- HATS: You 11H & Me 3H
- Model: 14H = 9H + 5H
- Top-level meaning:
  - 1. Correctly Adds up: 9 + 5 = 14
- Lower level, only formal:
  - 2. Correctly ordered: Y>M.
  - 3. Correct Parity: All odd.
  - 4. Correct Units: H.
- Still Wrong since not defined at atomic level: #H=?
- Decomposed Total H→Incorrectly model #H.

Decision Evaluation

- Attribution to **<u>Controllables</u>**:
  Evaluate the effects of decisions,
  as opposed to the effects of the market.
- Avoid arbitrariness
  Define these effects
  at the level of individual decisions.

Will here apply decision evaluation
to Risk Attribution.

Decision Process

- Choose process employed.
- Simple Single-Day EXAMPLE:
- Null: $R^B = \Sigma_S W^B{}_S \; R^B{}_S$
- Allocation: $R^A = \Sigma_S W^P{}_S \; R^B{}_S$
- Selection: $R^P = \Sigma_S W^P{}_S \; R^P{}_S$
- For now, Forgo the harder:
    - Multi-Period,
    - Nested &/Or
    - Forking Tree

DEFINE 'ATTRIBUTE'

The value of a controllable decision level attribute

≡ The effect on the parameter

of implementing the decision ≡

- The value of the parameter (e.g. IR)

after implementing the decision

Minus

- The value of the parameter

before implementing the decision.

This def. v. Standard PA for FI & multi: T, Curr, Asset, Nesting.

Decision Evaluation Ex.

- Chose Information Ratio
- $$IR^{P,B} \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)}$$
- Null: $IR^0 \equiv IR^{B,B} - IR^{B,B} = 0 - 0 = 0$
- Allocation: $IR^{All} \equiv IR^{A,B} - IR^{B,B} = IR^{A,B}$
- Selection: $IR^{Sel} \equiv IR^{P,B} - IR^{A,B} = IR^{P,B} - IR^{A,B}$

Total: $IR^{P,B}$

Decision Risk

The decision

To select holdings within sectors

Over the period of days

Increased the portfolio's IR by

$$IR^{Sel} = IR - IR^{A,B} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)}$$

□

□ = The IR attributed to Selection.

Decision Evaluation V. Decomposition Cont.

$$IR^S = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)} = \text{Effect of Sel on IR.}$$

Answers the precise Economic question:

How much did the implementation of 'Sel' decision increase the information ratio?

Whereas Decompostion's

$$IR\_Cont_A = \frac{\langle R^P - R^A \rangle}{\sigma(R^P - R^B)} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^P - R^B)}$$

Is an economically ill-formed "measure"

Constructed by an arbitrary formal process and then Named <u>"Contribution of Selection to IR."</u>

Misinformation from Standard Decomposition

If R^P tracked R^B

But R^A did not track R^B at all,

Then

Standard Decomposition results can imply that

Selection significantly decreased IR

When in fact,

As Decision Evaluation precisely shows,

Selection could have significantly increased IR.

Summary

- Standard methods of Decomposition
  - Only fully apply meaningful restrictions
    at the portfolio level,
    & try to obtain meaning at the decision or issue level.
  - Leading to formally consistent but misleading results.
- Decision Evaluation
  - Provides uniquely appropriate answers
    to precisely meaningful economic questions
    at the level sought.

Warning!

Standard Decomposition Methods

can Dangerously Misguide

any Investment Processed that relies on knowing:

- Issue-, Sector-, CarveOut-level trade-inclusive returns,
- Performance Attribution of Brinson-type,
- Contribution (W*R) or
- Risk Attribution.

Use Economically Meaningful Methods.

Decision Evaluation→Meaningful Results.